MENIIT
NEET | IIT-JEE | FOUNDATION
Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAIN-2020

COMPUTER BASED TEST (CBT)

DATE : 02-09-2020 (SHIFT-2) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART-A : PHYSICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

1. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is :

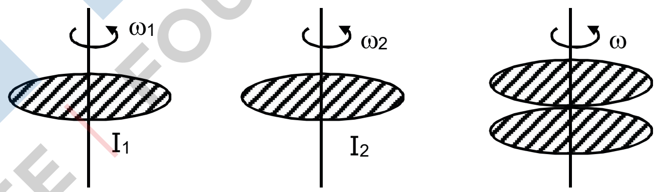
(1) $\frac{20}{3}$ J (2) $\frac{5}{3}$ J (3) $\frac{10}{3}$ J (4) $\frac{2}{3}$ J

Ans. (1)

Sol. Common angular velocity from angular momentum conversation

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{0.1\times 10 + 0.2\times 5}{0.1+0.2} = \frac{20}{3}$$

Final K.E.



$$K_f = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2$$

$$= \frac{1}{2}(0.1+0.2)\times\left(\frac{20}{3}\right)^2 \Rightarrow k_f = \frac{20}{3}$$

2. When the temperature of metal wire is increased from 0ºC to 10ºC, its length increases by 0.02%. The percentage change in its mass density will be closed to :

(1) 0.8 (2) 0.008 (3) 2.3 (4) 0.06

Ans. (4)

Sol. $\Delta\ell = \ell\alpha\Delta t$

$$\alpha = \frac{\Delta\ell}{\ell\Delta T} = \frac{0.02}{100\times 10}; \quad \alpha = 2\times 10^{-5}$$

$\gamma = 3\alpha = 6\times 10^{-5}$

$$\frac{\Delta V}{V} = \gamma\times\Delta T; \quad \frac{\Delta V}{V}\times 100 = (6\times 10^{-5}\times 10\times 100) = 6\times 10^{-2}$$

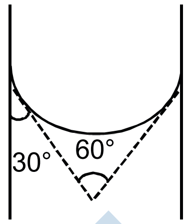
Volume increase by 0.06% therefore density decrease by 0.06%.

3. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to ($g = 10\ ms^{-2}$)

(1) 0.087 m (2) 0.137 m (3) 0.172 m (4) 0.049 m

Ans. (1)

Sol. $h = \frac{2T\cos\theta}{\rho g r}$ $\theta = 30°$



P = 667 kg/m^3; T = 1/20 Nm^{-1}; r = 0.15 × 10^{-3}

$$= \frac{2 \times \frac{1}{20} \times \frac{\sqrt{3}}{2}}{667 \times 10 \times 0.15 \times 10^{-3}} = 0.087\ m$$

4. The figure shows a region of length 'ℓ' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity $4\times10^5\ ms^{-1}$ making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, 'ℓ' is close to (mass of proton = 1.67×10^{-27} kg, charge of the proton = 1.6×10^{-19} C)

(1) 0.22 m (2) 0.88 m (3) 0.44 m (4) 0.11 m

Ans. (3)

Sol. ℓ = 10 × pitch

$$= 10 \times v\cos 60° \times \frac{2\pi m}{qB}$$

$$\ell = \frac{10\pi m v}{qB}$$

Put in the value of given data we find ℓ = 0.44

5. A heat engine is involved with exchange of heat of 1915 J, –40J, +125 J and –QJ, during one cycle achieving and efficiency of 50.0%. The value of Q is :

(1) 400 J (2) 980 J (3) 640 J (4) 40 J

Ans. (2)

Sol. **Ist case when Q_4 is negative**

$$\eta = \frac{W}{\Sigma Q_+} = \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3} = 0.5$$

$$\Rightarrow \quad \frac{1915 - 40 + 125 + Q_4}{1915 + 125} = 0.5$$

$\Rightarrow$ 1915 – 40 + 125 + Q_4 = 1020

$\Rightarrow$ Q_4 = 1020 – 2000 $\Rightarrow$ Q_4 = – Q = –980 J

Q = 980 J

IInd case when Q_4 is Positive

$$\frac{W}{\Sigma Q_+} = \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3 + Q_4} = 0.5$$

Q_4 = –1960

6. In a hydrogen atom the electron makes a transition from (n + 1)th level to the nth level. If n >> 1, the frequency of radiation emitted is proportional to :

(1) $\frac{1}{n^4}$ (2) $\frac{1}{n}$ (3) $\frac{1}{n^3}$ (4) $\frac{1}{n^2}$

Ans. (3)

Sol. $E_n = -\dfrac{Rhc}{n^2}$

$E_{n+1} = -\dfrac{Rhc}{(n+1)^2}$

$\Delta E = E_{n+1} - E_n$

$h\nu = Rhc\left[\dfrac{1}{n^2} - \dfrac{1}{(n+1)^2}\right]$

$\nu = R.c\left[\dfrac{(n+1)^2 - n^2}{n^2(n+1)^2}\right]$

$\nu = R.c\left[\dfrac{1+2n}{n^2(n+1)^2}\right]$

$n >> 1 \Rightarrow \qquad \nu = R.c\left[\dfrac{2n}{n^2 \times n^2}\right] = \dfrac{2RC}{n^3}$

$\nu \propto \dfrac{1}{n^3}$

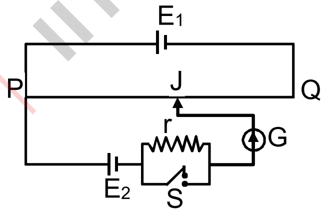
7. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be :

(1) 24 (2) 18 (3) 28 (4) 30

Ans. (3)

Sol. $y = \dfrac{m_1 D\lambda_1}{d} = \dfrac{mD\lambda_2}{d}$

8. A potentiometer wire PQ of 1m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is :



(1) 0.01 V/cm (2) 0.04 V/cm (3) 0.03 V/cm (4) 0.02 V/cm

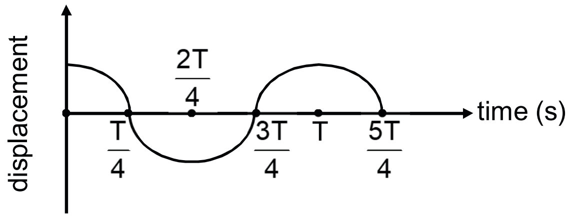
Ans. (4)

Sol. $X = V/\ell$

$x = \dfrac{1.02}{49}$

= 0.02 volt/cm

9. The displacement time graph of a particle executing SHM is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion ?

(A) The force is zero at $t = \frac{3T}{4}$

(B) The acceleration is maximum at t = T

(C) The speed is maximum at $t = \frac{T}{4}$

(D) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(1) (B), (C) and (D) (2) (A) and (D) (3) (A), (B) and (D) (4) (A), (B) and (C)

Ans. (4)

Sol. From graph equation of SHM.

X = A cosωt

(A) at $\frac{3T}{4}$ particle at mean position

$\therefore$ a = 0

F = 0

(B) at T particle again at extreme position so acceleration is maximum

(C) at $t = \frac{T}{4}$, particle is at mean position so velocity maximum.

$\therefore$ a = 0

(D) KE = PE

$$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$$

$A^2 = 2x^2$

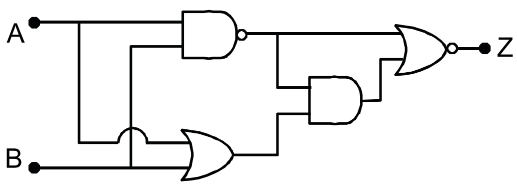
$$x = \frac{+A}{\sqrt{2}}$$

$$\frac{A}{\sqrt{2}} = A\cos\omega t$$

t = T/8

$\therefore$ A, B and C are correct

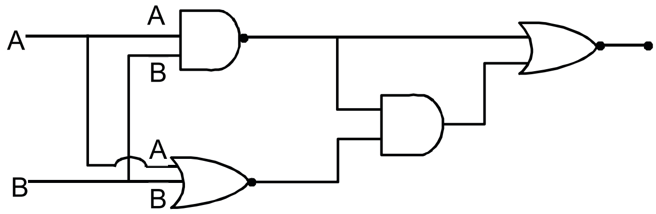
10. In the following, digital circuit, what will be the output a 'Z', when the input (A,B) are (1,0), (0,0), (1,1), (0,1)



(1) 1,1,0,1 (2) 0,1,0,0 (3) 1,0,1,1 (4) 0,0,1,0

Ans. (4)

Sol.



A	B	$\overline{A.B}$	$\overline{A+B}$	$P=\left(\overline{A.B}\right)\cdot\left(\overline{A+B}\right)$	$Q=P+\left(\overline{A.B}\right)$	$\overline{Q}=X$
1	0	1	0	0	1	0
0	1	1	0	0	1	0
1	1	0	0	0	0	1
0	0	1	1	1	0	0

$\therefore$ option (i) is the Answer

11. In momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is :

(1) $[P^2 AT^{-2}]$ (2) $[P^{-1} AT^{-1}]$ (3) $[PA^{1/2}T^{-1}]$ (4) $[PA^{-1}T^{-2}]$

Ans. (3)

Sol. Let dimension formula of energy will be

$E = A^a T^b P^c$

$P = M^1L^1T^{-1}$

$A = L^2$

$T = T^1$

$M^1L^2T^{-2} = M^cL^{2a+c}T^{b-c}$

by comparison

$C = 1$ .....(1)

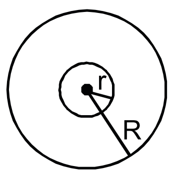
$2a + c = 2$ .....(2)

$b - c = -2$ .....(3)

$c = 1,\ a = 1/2,\ b = -1$

$E = A^{1/2}\ T^{-1}P^1$

12. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R (R > r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is :



(1) $\frac{1}{4\pi\epsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$ (2) $\frac{1}{4\pi\epsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$ (3) $\frac{1}{4\pi\epsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$ (4) $\frac{1}{4\pi\epsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$

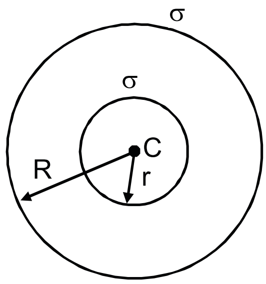
Ans. (4)

Sol. $Q_1 = \sigma 4\pi r^2$

$Q_2 = \sigma 4\pi R^2$

$\therefore \quad Q = \sigma 4\pi(r^2 + R^2)$

$\therefore \quad \sigma = \dfrac{Q}{4\pi(r^2 + R^2)}$



$V_C = \dfrac{KQ_1}{r} + \dfrac{KQ_2}{R}$

$= \dfrac{K\sigma 4\pi r^2}{r} + \dfrac{K\sigma 4\pi R^2}{R}$

$= K\sigma 4\pi(r + R)$

$= \dfrac{KQ4\pi(r + R)}{4\pi(r^2 + R^2)}$

$= \dfrac{KQ(r + R)}{(r^2 + R^2)}$

13. In plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i} - 2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave.

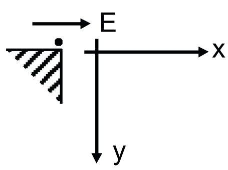
(1) $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ (2) $\frac{1}{\sqrt{5}}(\hat{i} + 2\hat{j})$ (3) $\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$ (4) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$

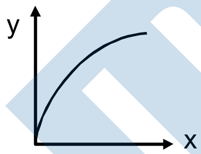
Ans. (4)

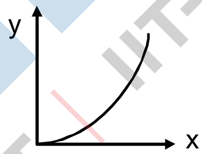
Sol. $\vec{E} \times \vec{B} \,\square\, \vec{C}$

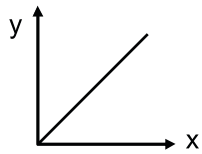
$\vec{E} \times \vec{B} = \dfrac{1}{\sqrt{2}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \dfrac{\hat{i} + \hat{j}}{\sqrt{2}} \qquad \Rightarrow \vec{C} = \dfrac{\hat{i} + \hat{j}}{\sqrt{2}}$

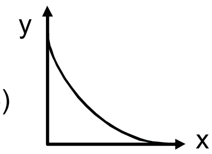
14. A small point mas carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass ? (Curves are drawn schematically and are not to scale)



(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. $\vec{F} = qE\hat{i} + mg\hat{j}$

Since initial velocity is zero. It will move in straight line.

15. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true ?

(A) the mean free path of the molecules decreases

(B) the mean collision time between the molecules decreases.

(C) the mean free path remains unchanged.

(D) the mean collision time relations unchanged.

(1) (A) and (D) (2) (A) and (B) (3) (B) and (C) (4) (C) and (D)

Ans. (3)

Sol. As we know mean free path

$$\lambda = \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$$

N = no. of molecule

V = volume of container

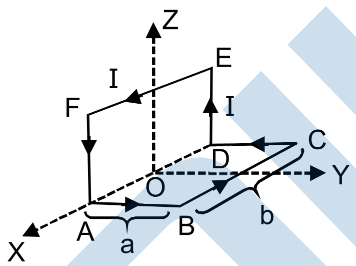
d = diameter of molecule

Velocity constant and no. of molecules are same.

So mean free path remains same.

As temperature increases no. of collision increases so relaxation time decrease.

16. A carrying current I is bent in the shape ABCDEFA as shown, where rectangle ABCEA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is :



(1) $\sqrt{2}abI$ along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$ (2) $\sqrt{2}abI$ along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{\hat{k}}{\sqrt{5}}\right)$

(3) abI, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$ (4) abI, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

Ans. (1)

Sol. Loop ABCD

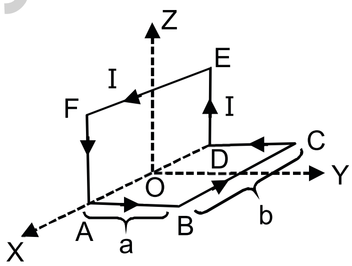
$\vec{M}_1 = (abi)(\hat{k})$

For Loop DEFA

$\vec{M}_2 = (abi)(\hat{j})$

$\vec{M} = \vec{M}_1 + \vec{M}_2$; $\vec{M} = (abi)(\hat{j}+\hat{k})$

$\sqrt{2}abI$ along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$



17. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to :

(1) 1.2×10^{-28} kg (2) 9.7×10^{-28} kg (3) 9.1×10^{-31} kg (4) 4.8×10^{-27} kg

Ans. (2)

Sol. $\lambda = \frac{h}{mv}$

$$\Rightarrow \quad \frac{m'}{m} = \frac{v\lambda}{v'\lambda'} = \frac{1}{5} \times \frac{1}{1.878} \times 10^{-4} \times 9.1 \times 10^{-31}$$

$m' = 9.7 \times 10^{-28}$ kg

18. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected)

(1) R/2 (2) $\frac{\sqrt{5}R - R}{2}$ (3) $\frac{\sqrt{5}}{2}R - R$ (4) $\frac{\sqrt{3}R - R}{2}$

Ans. (2)

Sol. $\frac{GM}{(R+h)^2} = \frac{GM}{R^3}(R-h)$

$R^3 = (R + h)^2 (R - h)$

$= (R^2 + h^2 + 2hR) (R - h)$

$R^3 = R^3 + h^2R + 2hR^2 - R^2h - h^3 - 2h^2R$

$h^3 + h^2 (2R - R) - R^2h = 0$

$h^3 + h^2R - R^2h = 0$

$h^2 + hR - R^2 = 0$

$$h = \frac{-R \pm \sqrt{R^2 + 4(1)R^2}}{2}$$

$$= \frac{-R + \sqrt{5}R}{2}$$

$$= \frac{(\sqrt{5} - 1)}{2}R$$

19. A 10μF capacitor is fully to a potential difference of 50VAfter removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is :

(1) 10μF (2) 20μF (3) 30μF (4) 15μF

Ans. (4)

Sol. $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$20 = \frac{10 \times 50 + 0}{20 + C}$$

C = 15μF

20. An inductance coil has a reactance of 100Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is :

(1) 5.5×10^{-5} H (2) 1.1×10^{-2} H (3) 6.7×10^{-7} H (4) 1.1×10^{-1} H

Ans. (2)

Sol. $\tan\theta = \frac{x_L}{R} = \tan 45°$

$x_L = R$

$= 100 = \sqrt{X_L^2 + R^2}$

$100 = \sqrt{R^2 + R^2}$

$\sqrt{2}R = 100$

$R = 50\sqrt{2}$

$\therefore X_L = 50\sqrt{2}$

$L\omega = 50\sqrt{2}$

$L = \dfrac{50\sqrt{2}}{2\pi \times 1000} = \dfrac{25\sqrt{2}}{\pi}$mH.

= 1.1 × 10^{-2} H

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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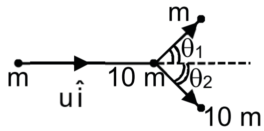
21. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is ________.



Ans. 10.00

Sol. From momentum conservation in perpendicular direction of initial motion.

$mv_1 \sin\theta_1 = 10mv_2 \sin\theta_2$ ......(1)

Given that $\left(\frac{1}{2}mu^2\right)\frac{1}{2} = \frac{1}{2}mv_1^2$ ....(2)

From equation (1) & (2)

$\sin\theta_1 = \sqrt{10}\sin\theta_2$

n = 10

22. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire (Young's modulus of wire Y = 9 × 1010 Nm^{-2}), (to the nearest integer),________

Ans. 35.00

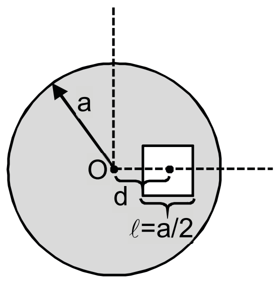
Sol. Fundamental frequency in the string

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{\rho A}} = \frac{1}{2\ell}\frac{\sqrt{Y\Delta\ell}}{\rho\ell}$$

$$f = \frac{1}{2\ell}\sqrt{\frac{Y\Delta\ell}{\rho\ell}} \qquad \left(\frac{\Delta\ell}{\ell} = 4.9 \times 10^{-4}\right)$$

$$= \frac{1}{2\ell}\sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9 \times 10^{-3} \times 10^{-6}}} = 35.$$

23. A square shaped hole of side $\ell = a/2$ is carved out at a distance d = a/2 from the centre O of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is :

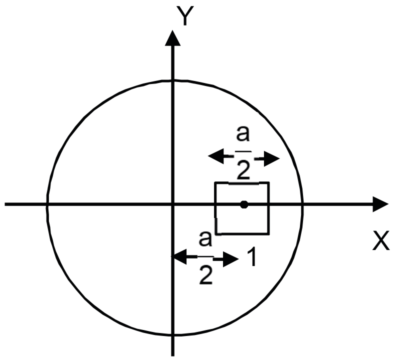


Ans. 23.00

Sol. $X_a = \dfrac{A.x - A.x1}{A - A_1} = \dfrac{\pi a^2 \times 0 - \dfrac{a^2}{4} \times \dfrac{a}{2}}{\pi a^2 - \dfrac{a^2}{4}}$

$= \dfrac{-a^3/8}{\left(\pi - \dfrac{1}{4}\right)a^2} = \dfrac{-a}{2(4\pi - 1)} = \dfrac{-a}{8\pi - 2} = -\dfrac{a}{23}$

x = 23



24. A light ray enters a solid glass sphere of refractive index m = $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is _____________.

Ans. 90.00

Sol. Apply Snell's law at S_1

$1 \sin 60° = \sqrt{3} \sin r$

$\sin r = \dfrac{1}{2}$

r = 30°

from geometry

r' = 30°

Again apply snell's law on S_2

$\sqrt{3} \sin r' = 1 \sin e$

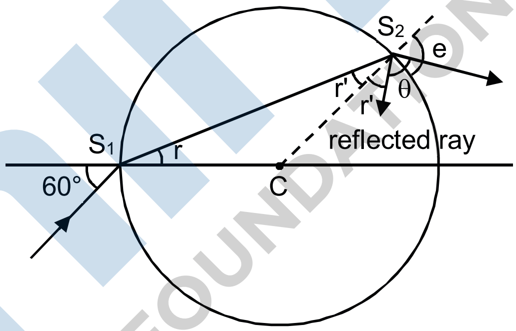
$\dfrac{\sqrt{3}}{2} = \sin e$

$\therefore$ e = 60°

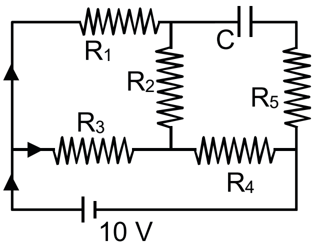
from geometry

r' + θ + e = 180°

θ = 90°



25. An ideal cell of emf 10V is connected in circuit shown in figure. Each resistance is 2Ω. The potential difference (in V) across the capacitor when it is fully charged is _____________

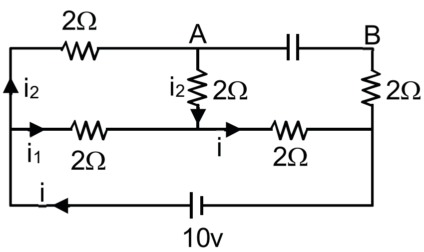


Ans. 08.00

Sol. $i = \dfrac{10}{\dfrac{4}{3} + 2} = \dfrac{10 \times 3}{10} = 3$ Amp

i_1 = 2A & i_2 = 1A

V_{AB} = 1 × 2 + 3 × 2 = 8 V



PART-B : CHEMISTRY

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

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26. Cast iron is used for the manufacture of :

(1) wrought iron and steel
(2) wrought iron, pig iron and steel
(3) wrought iron and pig iron
(4) pig iron, scrap iron and steel

Ans. (1)

Sol. Cast iron is made from pig iron which is used for production of wrought iron & steel.

27. Amongst the following statements regarding adsorption, those that are valid are:

(a) ΔH becomes less negative as adsorption proceeds.

(b) On a given adsorbent, ammonia is adsorbed more than nitrogen gas.

(c) On adsorption, the residual force acting along the surface of the adsorbent increases.

(d) With increase in temperature, the equilibrium concentration of adsorbate increases.

(1) (b) and (c)
(2) (c) and (d)
(3) (a) and (b)
(4) (d) and (a)

Ans. (3)

Sol. (a) When gas is adsorbed on metal surface.

ΔH become less negative with progress of reaction.

(b) Gas with greater value of critical temperature (T_C) absorbed more. As $T_C(NH_3) > TC(N_2)$

So NH_3 absorbed more than N_2.

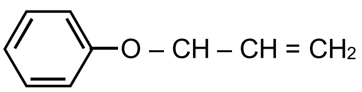
28. The size of a raw mango shrinks to a much smaller size when kept in a concentrated salt solution. Which one of the following process can explain this ?

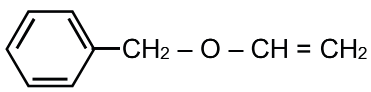
(1) Dialysis
(2) Reverse osmosis
(3) Osmosis
(4) Diffusion

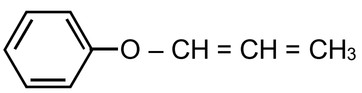
Ans. (3)

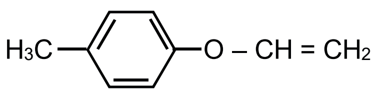
Sol. When mango kept in concentrate salt solution then solvent (water) flow from mango to concentrate solution that's why mango shrinks this is called. "Osmosis"

29. An organic compound 'A' ($C_9H_{10}O$) when treated with conc. HI undergoes cleavage to yield compound 'B' and 'C'. 'B' gives yellow precipitate with AgNO3 where as 'C' tautomerizes to 'D'. 'D' gives positive iodoform test. 'A' could be:

(1) 

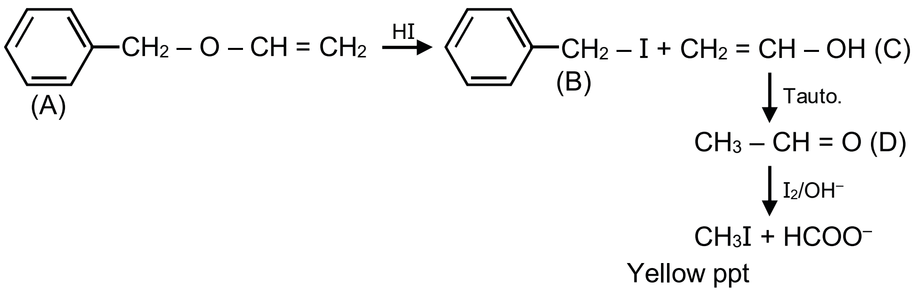
(2) 

(3) 

(4) 

Ans. (2)

Sol. $C_9H_{10}O$



30. The number of subshells associated with n = 4 and m = –2 quantum numbers is:

(1) 16 (2) 8 (3) 4 (4) 2

Ans. (4)

Sol. For n= 4 possible values of ℓ = 0, 1, 2, 3 only ℓ = 2 & ℓ = 3 can have m = –2.

So possible subshells are 2.

31. Two compounds A and B with same molecular formula (C_3H_6O) undergo Grignard reaction with methyl magnesium bromide to give products C and D. Products C and D show following chemical tests.

Test	C	D
Ceric ammonium nitrate Test	Positive	Positive
Lucas Test	Turbidity obtained after five minutes	Turbidity obtained immediately
Iodoform Test	Positive	Negative

C and D respectively are:

(1) C = $H_3C – CH_2 – CH(OH) – CH_3$; D = $H_3C – C(CH_3)_2 – OH$

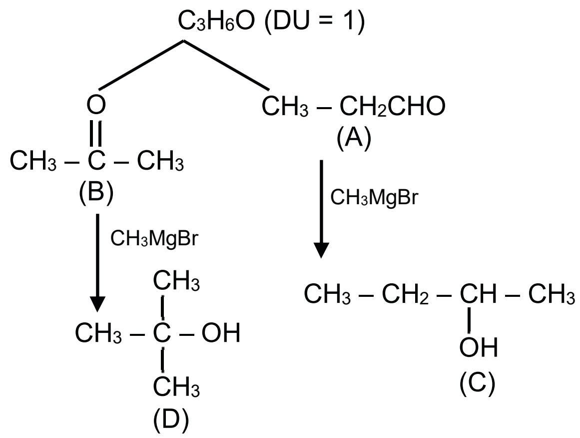
(2) C = $H_3C – CH_2 – CH_2 – CH_2 – OH$; D = $H_3C – C(CH_3)_2 – OH$

(3) C = $H_3C – C(CH_3)_2 – OH$; D = $H_3C – CH_2 – CH(OH) – CH_3$

(4) C = $H_3C – CH_2 – CH_2 – CH_2 –OH$; D = $H_3C – CH_2 – CH(OH) – CH_3$

Ans. (1)

Sol.



Iodoform Test	–ve	+ve
Lucas Test	Immediate	after 5-10 Mint.
Ceric Ammonium nitrate	+ve	+ve

32. Match the type of interaction in column A with the distance dependence of their interaction energy in column B :

A	B
(i) ion–ion (a)	(a) $\frac{1}{r}$
(ii) Dipole–dipole (b)	(b) $\frac{1}{r^2}$
(iii) London dispersion (c)	(c) $\frac{1}{r^3}$
	(d) $\frac{1}{r^6}$

(1) (i) – (a); (ii) – (b); (iii) – (c)
(2) (i) – (a); (ii) – (b); (iii) – (d)
(3) (i) – (b); (ii) – (d); (iii) – (c)
(4) (i) – (a); (ii) – (c); (iii) – (d)

Ans. (Reso Answer 4 Given NTA Answer 2)

Sol. (i) ion-ion interaction energy is inversely proportional to the distance between ions $\left(\frac{1}{r}\right)$.

(ii) dipole-dipole interaction energy is inversely proportional to the third power of r $\left(\frac{1}{r^3}\right)$.

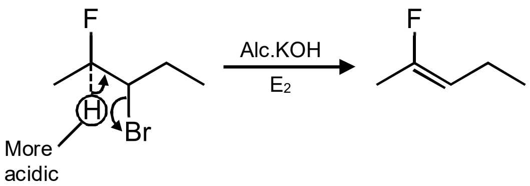
(iii) The interaction energy of London force is inversely proportional to sixth power of distance between two interaction particles $\left(\frac{1}{r^6}\right)$.

33. The major product obtained from E_2-elimination of 3-bromo-2- fluoropentane is

(1) $CH_3-CH_2-\overset{Br}{\overset{|}{C}}=CH-CH_3$
(2) $CH_3-CH_2-\overset{Br}{\overset{|}{C}}H-CH=CH_2$
(3) $CH_3-CH=CH-\overset{F}{\overset{|}{C}}H-CH_3$
(4) $CH_3-CH_2-CH=\underset{CH_3}{\underset{|}{C}}-F$

Ans. (4)

Sol.



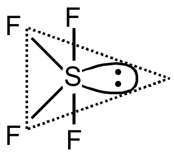
34. The molecular geometry of SF_6 is octahedral. What is the geometry of SF_4 (including lone pair(s) of electrons, if (any)?

(1) Square planar (2) Trigonal bipyramidal

(3) Tetrahedral (4) Pyramidal

Ans. (2)

Sol. $SF_4 \Rightarrow$ Steric No = 5 so hybridisation is sp^3d.



Geometry is trigonal bipyramidal but shape is "See Saw".

35. The results given in the below table were obtained during kinetic studies of the following reaction :

$$2A + B \longrightarrow C + D$$

Experiment	[A] /molL⁻¹	[B] /molL⁻¹	Initial rate /molL⁻¹ min⁻¹
I	0.1	0.1	6.00×10^{-3}
II	0.1	0.2	2.40×10^{-2}
III	0.2	0.1	1.20×10^{-2}
IV	X	0.2	7.20×10^{-2}

X and Y in the given table are respectively :

(1) 0.3, 0.4 (2) 0.4, 0.3 (3) 0.4, 0.4 (4) 0.3, 0.3

Ans. (1)

Sol. Rate = k $[A]^a [B]^b$

from Exp (1) & (2) b = 2

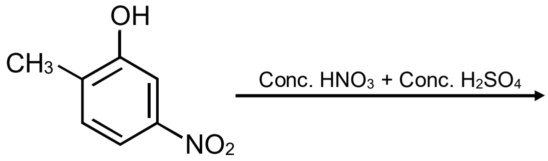
from Exp (1) & (3) a = 1

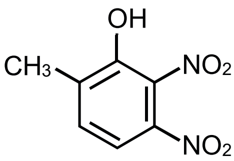
from Exp (2) & (4) $\Rightarrow 3 = \left(\frac{x}{0.1}\right)^1$ so x = 0.3

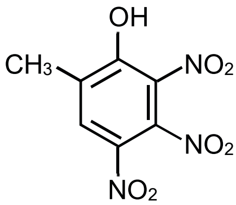
from Exp (1) & (5) $\Rightarrow 48 = (3)^1\left(\frac{y}{0.1}\right)^2$

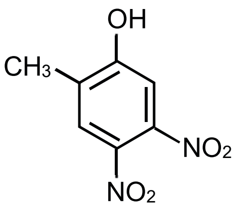
$$(4)^2 = \left(\frac{y}{0.1}\right)^2 \quad \text{so } y = 0.4$$

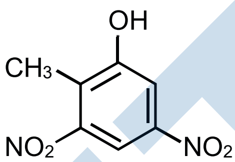
36. The major product of the following reaction is:



(1) 

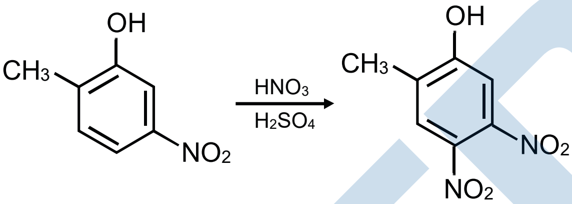
(2) 

(3) 

(4) 

Ans. (3)

Sol. This is electrophilic substitution reaction which is determine by electronic effect of $OH\backslash CH_3\backslash NO_2$.



37. The one that is not expected to show isomerism is:

(1) $[Ni(NH_3)_2Cl_2]$ (2) $[Ni(NH_3)_4(H_2O)_2]^{2+}$ (3) $[Ni(en)_3]^{2+}$ (4) $[Pt(NH_3)_2Cl_2]$

Ans. (1)

Sol. (1) $[\overset{+2}{Ni}(NH_3)_2Cl_2] \Rightarrow$ Ni^{2+} $\Rightarrow$ $3d^8 4s^0$.

$\Rightarrow$ sp^3 hybridisition.

$\Rightarrow$ tetrahedral.

so $[Ni(NH_3)_2Cl_2]$ do not show isomerism.

(2) $[Ni(NH_3)_4(H_2O)_2]^{2+}$, show geometrical isomerism.

(3) $[Ni(en)_3]^{2+}$, show optical isomerism.

(4) $[Pt(NH_3)_2Cl_2]$, show geometrical isomerism.

38. The shape/structure of $[XeF_5]^-$ and XeO_3F_2, respectively are :

(1) octahedral and square pyramidal (2) trigonal bipyramidal and trigonal bipyramidal

(3) trigonal bipyramidal and pentagonal (4) pentagonal planar and trigonal bipyramidal

Ans. (4)

Sol. (i) XeF_5^- St. No. = (5 + 2) = 7

so hybridisation is = sp^3d^3

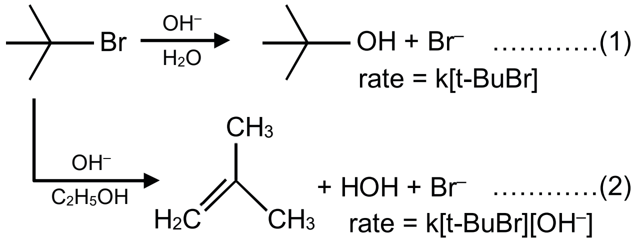
and structure is pentagonal planar.

(ii) XeO_3F_2 St. No. = 5

so hybridisation is = sp^3d

and structure is trigonal bipyramidal.

39. Consider the reaction sequence given below :



Which of the following statements is true:

(1) Changing the concentration of base will have no effect on reaction (1)

(2) Doubling the concentration of base will double the rate of both the reactions

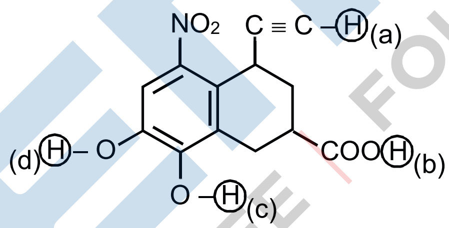
(3) Changing the concentration of base will have no effect on reaction (2)

(4) $^{\ominus}OR$ will have no effect on reaction (2)

Ans. (1)

Sol. First reaction is SN1 in which rate does not depend on conc. of nucleophile. Second reaction is E_2 reaction in which rate depends on conc. of base.

40. Arrange the following labelled hydrogens in decreasing order of acidity:



(1) b > c > d > a (2) c > b > a > d (3) b > a > c > d (4) c > b > d > a

Ans. (1)

Sol. Acidic strength ∝ Stability of conjugate base

General order of acidic strength

R–COOH > Ph–OH > R–C≡CH

'c' is more acidic due to –M effect of $-NO_2$.

41. If you spill a chemical toilet cleaning liquid on your hand, your first aid would be:

(1) vinegar (2) aqueous NaOH (3) aqueous $NaHCO_3$ (4) aqueous NH_3

Ans. (3)

Sol. In toilet cleaning liquid the main constituent is HCl, which can cause skin burn so it should be treated with $NaHCO_3$ which can easily consume the acid.

42. The correct observation in the following reaction is:

$$\text{Sucrose} \xrightarrow[\text{Cleavage (Hydrolysis)}]{\text{Gly cosidic bond}} A + B \xrightarrow[\text{Reagent}]{\text{Seliw an off}} ?$$

(1) Formation of blue colour (2) Formation of violet colour

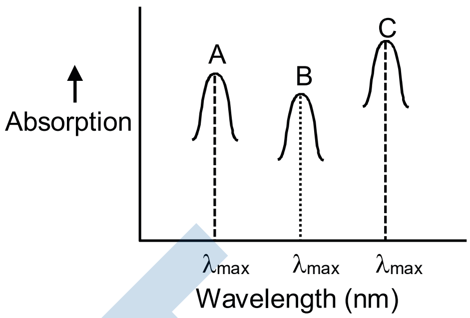
(3) Formation of red colour (4) Gives no colour

Ans. (3)

Sol. Seliwanoff reagent → [Resorcinol + Conc. HCl]

Use of Seliwanoff reagent is to distinguish aldoses and ketoses. Ketoses show red colour with Seliwanoff Reagent.

43. Simplified absorption spectra of three complexes ((i) and (ii) and (iii)) of M^{+n} ion are provided below; their λ_{max} values are marked as A, B and C respectively. The correct match between the complexes and their λ_{max} values is:



(i) $[M(NCS)_6]^{(-6+n)}$ (ii) $[MF_6]^{(-6+n)}$

(iii) $[M(NH_3)_6]^{n+}$

(1) A-(i), (B)-(ii), C-(iii)
(2) A-(ii), (B)-(iii), C-(i)
(3) A-(ii), (B)-(i), C-(iii)
(4) A-(iii), (B)-(i), C-(ii)

Ans. (4)

Sol. Stronger the ligand greater is splitting of d orbitals and smaller will be wave length of light absorbed.

The splitting power of ligands is $NH_3 < NC\bar{S} < F^-$

So order of wave length of light absorbed is $\lambda_{NH_3} < \lambda_{NC\bar{S}} < \lambda_{F^-}$

44. Two elements A and B have similar chemical properties. They don't form solid hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively, are :

(1) Li and Mg (2) Na and Ca (3) Na and Rb (4) Cs and Ba

Ans. (1)

Sol. Li and Mg do not form solid bicarbonate. But react with N_2 to give nitrides.

$$6\,Li + N_2 \xrightarrow{\Delta} 2\,Li_3N$$

$$3\,Mg + N_2 \xrightarrow{\Delta} Mg_3N_2.$$

45. Three elements X, Y and Z are in the 3rd period of the periodic table. The oxides of X, Y and Z, respectively are basic amphoteric and acidic. The correct order of the atomic numbers of X, Y and Z is:

(1) X < Y < Z (2) Z < Y < X (3) X < Z < Y (4) Y < X < Z

Ans. (1)

Sol. On moving left to right in a period.

Acidic character of oxides is increase.

3^{rd} period element oxides.

$\underbrace{Na_2O \quad MgO}_{Basic} \quad \underbrace{Al_2O_3}_{Amphoteric} \quad \underbrace{Si_2O \quad P_2O_5}_{Acidic}$

(i) Acidic character↑

(i) Atomic No↑

So X have minimum Atomic No

& Z have maxima Atomic No

So correct order is X < Y < Z

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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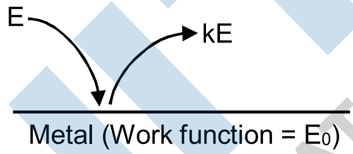
46. The work function of sodium metal is 4.41×10^{-19} J. If photons of wavelength 300 nm are incident on the metal, the kinetics energy of the ejected electrons will be ($h = 6.63 \times 10^{-34}$Js; $c = 3 \times 10^{8}$ m/s) _____$\times 10^{-21}$ J.

Ans. 222

Sol. $E = E_0 + (kE)_{max}$



$$\frac{hC}{\lambda} = 4.41 \times 10^{-19} + kE$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 108}{300 \times 10^{-9}} = 4.41 \times 10^{-19} + kE$$

So, $(kE)_{max} = 6.63 \times 10^{-19} - 4.41 \times 10^{-19}$

$= 2.22 \times 10^{-19}$

$= 222 \times 10^{-21}$J

47. The ration of the mass percentages of 'C & H' and 'C & O' of a saturated acyclic organic compound 'X' are 4 : 1 and 3 : 4 respectively. Then, the moles of oxygen gas required for complete combustion of two moles of organic compound 'X' is_________.

Ans. 5

Sol. Mass ratio of C : H is 4 : 1 $\Rightarrow$ 12 : 3

& C : O is 3 : 4 $\Rightarrow$ 12 : 16

		Mass	mol	mol ratio
so	C	12	1	1
	H	3	3	3
	O	16	1	1

Empirical formula $\Rightarrow CH_3O$

as compound is saturated acyclic so molecular formula is $C_2H_6O_2$.

$$C_2H_6O_2 + \frac{5}{2}O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(g)$$

mol 2 mol 5 mol

so required moles of O_2 is $\Rightarrow$ 5

48. The heat of combustion of ethanol into carbon dioxides and water is –327 kcal at constant pressure. The heat evolved (in cal.) at constant volume at 27°C (if all gases behave ideally) is (R = 2 cal mol^{-1} K^{-1}) ________.

Ans. –326400

Sol. $C_2H_5OH(\ell) + O_2(g) \longrightarrow 2O_2(g) + 3H_2O(\ell)$; $\Delta HC = -327$ Kcal

$\Delta H_C = \Delta U_C + \Delta n_g RT$

$-327 \times 10^3 = \Delta UC + 1 \times 2 \times 300$

$\Delta UC = -326400$ cal

So heat evolved as constant volume is –326400 cal

49. The oxidation states of transition metal atoms in $K_2Cr_2O_7$, $KMnO_4$ and K_2FeO_4, respectively, are x, y and z. The sum of x, y and z is_______.

Ans. 19

Sol.

	Compound	Oxidation state of transition element.
(i)	$K_2Cr_2O_7$	x = +6
(ii)	$KMnO_4$	y = +7
(iii)	K_2FeO_4	z = +6

so (x + y + z) = 19

50. For the disproportionation reaction $2Cu^{+}(aq) \rightleftharpoons Cu(s) + Cu^{2+}(aq)$ at 298 K, ln K (where K is the equilibrium constant) is________ × 10^{-1}.

Given : $\left(E^{o}_{Cu^{2+}/Cu^{+}} = 0.16V\ E^{o}_{Cu^{+}/Cu} = 0.52V\ \frac{RT}{F} = 0.025\right)$

Ans. 144

Sol. $E^{o}_{cell} = E^{o}_{Cu^{+}/Cu} - E^{o}_{Cu^{2+}/Cu^{+}}$

$= 0.52 - 0.16$

$= 0.36$ V

$$\boxed{E^{o}_{cell} = \frac{RT}{nF} \ln K_{eq}}$$

$$0.36 = \frac{0.025}{1} \ln k$$

ln k = 14.4

$= 144 \times 10^{-1}$

PART-C : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

51. Let S be the sum of the first 9 term of the series :

$(x + ka\} + (x^2 + (k + 2)a\} + \{x^3 + (k + 4)a\} + \{x^4 + (k + 6)a\} + \ldots\ldots$ where $a \neq 0$ and $x \neq 1$.

If $S = \dfrac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to

(1) – 3 (2) 1 (3) – 5 (4) 3

Ans. (1)

Sol. $\dfrac{x(x^9-1)}{x-1} + a.\dfrac{9}{2}[2k + 8.2] = \dfrac{x^{10} - x + 9.a(k+8)(x-1)}{(x-1)}$

so 45 = 9 (k + 8) $\Rightarrow$ k = – 3

52. The imaginary part of

$\left(3 + 2\sqrt{-54}\right)^{\frac{1}{2}} - \left(3 - 2\sqrt{-54}\right)^{\frac{1}{2}}$ can be

(1) $\sqrt{-6}$ (2) $\sqrt{6}$ (3) $-2\sqrt{6}$ (4) 6

Ans. (3)

Sol. $\left|3 + 2\sqrt{-54}\right| = \sqrt{9 + 216} = 15$

$\Rightarrow \left(3 + 2\sqrt{-54}\right)^{1/2} = \pm\left(\sqrt{\dfrac{15+3}{2}} + i\sqrt{\dfrac{15-3}{2}}\right)$

$= \pm\left(3 + i\sqrt{6}\right)$

and $\left(3 - 2\sqrt{-54}\right)^{1/2} = \pm\left(3 + i\sqrt{6}\right)$

Hence $\left\{\left(3 + 2\sqrt{-54}\right)^{1/2} - \left(3 - 2\sqrt{-54}\right)^{1/2}\right\}$

$= \pm 2i\sqrt{6}$ or ± 6

Hence imaginary part $= \pm 2\sqrt{6}$

53. A plane passing through the point (3,1,1) contains two lines whose direction ratios are 1, – 2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α, –3, 5), then α is equal to

(1) 5 (2) 10 (3) –5 (4) –10

Ans. (1)

Sol. Plane is a(x – 3) + b(y – 1) + c (z – 1) = 0

dr's lines are (1, –2, 2) + (2,3 – 1) $\therefore$ dr's of normal of plane (4, – 5, – 7)

$\therefore$ plane in 4x – 5y –7z = 0

(α, – 3, 5) lies on plane

$\therefore$ α = 5

54. The equation of the normal to the curve $y = (1 + x)^{2y} + \cos^2 (\sin^{-1} x)$ at x = 0 is :

(1) y = 4x + 2 (2) y + 4x = 2 (3) x + 4y = 8 (4) 2y + x = 4

Ans. (3)

Sol. $y = (1 + x)^{2y} + \cos^2(\sin^{-1} x)$

x = 0, y = 2

$y = e^{2y\ell(1+x)} + (1 - x^2)$

$$\frac{dy}{dx} = e^{2y\ell n(1+x)}\left\{\frac{2y}{1+x} + \ell n(1+x).2y'\right\} - 2x$$

$$y' = \left(\frac{2\times 2}{1+0} + 0\right)$$

y' = 4

y – 2 = –1/4 (x – 0)

4y – 8 = –x

x + 4y = 8

55. $\lim\limits_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to

(1) 2 (2) 1 (3) e (4) e^2

Ans. (4)

Sol. $e^{\lim\limits_{x\to 0}\frac{1}{x}\left(\tan\left(\frac{\pi}{4}+x\right)-1\right)} = e^{\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{1+\tan x}{1-\tan x}-1\right)} \Rightarrow e^{\lim\limits_{x\to 0}\frac{2\tan x}{(1-\tan x)}} = e^{\lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)\left(\frac{2}{1-\tan x}\right)}$

$\Rightarrow e^{(1)\left(\frac{2}{1-0}\right)} = e^2$

56. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2 \sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse , $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is

(1) $\frac{2\sqrt{5}}{3}$ (2) $2\sqrt{6}$ (3) $\frac{4\sqrt{5}}{3}$ (4) $\sqrt{30}$

Ans. (3)

Sol. $\frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1 \Rightarrow e_H = \sqrt{1+\cos^2\theta}$ and $\frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1 \Rightarrow e_E = \sqrt{1-\cos^2\theta} = \sin\theta$

as given $e_H = \sqrt{5}\ e_E$

$\Rightarrow 1 + \cos^2\theta = 5\sin^2\theta \quad \Rightarrow \quad \cos^2\theta = 2/3$

Now length of L.R. of ellipse $= \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$

57. Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \to q$ (2) $(q \to p) \vee \sim (p \to q)$

(3) $(\sim q) \vee (p \wedge q) \to q$ (4) $(p \to q) \wedge (q \to p)$

Ans. (1)

Sol. (i) $\sim p \wedge (p \vee q) \to q$

$(\sim p \wedge p) \vee (\sim p \wedge q) \to q$

$C \vee (\sim p \wedge q) \to q$

$(\sim p \wedge q) \to q$

$\sim(\sim p \wedge q) \vee q$

$= (p \vee \sim q) \vee q = p \vee t = t$

(iii) $(\sim q) \vee (p \wedge q) \to q$ use $\sim(p \to q) = p \wedge \sim q \Rightarrow p \to q = \sim p \vee q$

$= (\sim q \vee p) \wedge (\sim q \vee q) \to q$

$= (\sim q \vee p) \to q$

$= (q \wedge \sim p) \vee q$

$= q$

(ii) and (iv)

P	q	$p \to q$	$q \to p$	$\sim (p \to q)$	$(p \to q) \wedge (q \to p)$	$(q \to p) \vee \sim (p \to q)$
T	T	T	T	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

58. Let EC denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$ then $\left(E_2^C \cap E_3^C / E_1\right)$ is equal to

(1) $P\left(E_3^C\right) - P(E_2)$ (2) $P\left(E_3^C\right) - P(E_2^C)$ (3) $P(E_3) - P(E_2^C)$ (4) $P(E_2^C) + P(E_3)$

Ans. (1)

Sol. $P((B^C \cap C^C) / A) = \frac{P(A \cap (B^C \cap C^C))}{P(A)}$

$= \frac{P(A) - \{P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)\}}{P(A)}$

$= \frac{P(A) - P(A).P(B) - P(A).P(C) + 0}{P(A)}$

$= 1 - P(B) - P(C)$

$= P(C^C) - P(B)$ or $P(B^C) - P(C)$

59. Let A = {x = (x, y, z)^T : PX = 0 and $x^2 + y^2 + z^2 = 1$}, where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$ then the set A :

(1) is a singleton. (2) contains more than two elements

(3) contains exactly two elements (4) is an empty set.

Ans. (3)

Sol. $\because$ |P| = 0

$\therefore$ system of equations

x + 2y + z = 0

2x – 3y + 4z = 0

x + 9y – z = 0 has

infinitely many solution let z = k $\in$ R

then $x = -\frac{11k}{7}, y = \frac{2k}{7}, z = k$

but $x^2 + y^2 + z^2 = 1$

$\therefore K = \pm\frac{7}{\sqrt{174}}$

$\therefore$ two solutions only

60. If a curve y = f(x), passing through the point (1,2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)\,dx$, then $f\left(\frac{1}{2}\right)$ is equal to

(1) $\frac{1}{1-\log_e 2}$ (2) 1 + loge^2 (3) $\frac{1}{1+\log_e 2}$ (4) $\frac{-1}{1+\log_e 2}$

Ans. (3)

Sol. $\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{2x^2} \Rightarrow y^{-2}\frac{dy}{dx} - \frac{1}{y}\cdot\frac{1}{x} = \frac{1}{2x^2}$

Put $-\frac{1}{y} = t \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dt}{dx} + \left(\frac{1}{x}\right)t = \frac{1}{2x^2}$

Linear differential equation

I.F.$e^{\int\frac{1}{x}dx} = e^{\ell nx} = x$

so solution of the linear differential equation is

$tx = \int\frac{1}{2x^2}.xdx + C \Rightarrow -\frac{x}{y} = \frac{1}{2}\ell nx + C$

The curve passes through (1,2)

$\Rightarrow \frac{1}{2} = \frac{1}{2}\ell n1 + C \Rightarrow C = -\frac{1}{2}$

Hence $-\frac{x}{y} = \frac{1}{2}\ell nx - \frac{1}{2}$

or $\frac{x}{y}=\frac{1-\ell nx}{2} \Rightarrow y=\frac{2x}{1-\ell nx}$ $\Rightarrow f\left(\frac{1}{2}\right)=\frac{2x\frac{1}{2}}{1-\ell n\frac{1}{2}}=\frac{1}{1+\ell n2}$

61. Let a, b, c $\in$ R be all non-zero satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

$A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ stratifies $A^T A = I$, then a value of abc can be :

(1) $\frac{1}{3}$ (2) $-\frac{1}{3}$ (3) 3 (4) $\frac{2}{3}$

Ans. (1)

Sol. $A^TA=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}=\begin{bmatrix} a^2+b^2+c^2 & ab+bc+ac & ab+bc+ac \\ ab+bc+ac & a^2+b^2+c^2 & ab+bc+ac \\ ab+bc+ac & ab+bc+ac & a^2+b^2+c^2 \end{bmatrix}=I$

So $\Sigma a^2 = 1$ and $\Sigma ab = 0$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

$= (a + b + c). 1$

$=\sqrt{\Sigma a^2+3\Sigma ab}$

$=\sqrt{1+0}$

$\Rightarrow 2 - 3abc = 1$

$\Rightarrow 3abc = 1$ $\Rightarrow abc=\frac{1}{3}$

62. If the equation $\cos 4\theta + \sin 4\theta + \lambda = 0$ has real solutions for θ, then λ lies in interval :

(1) $\left(-\frac{5}{4},-1\right)$ (2) $\left[-1,-\frac{1}{2}\right]$ (3) $\left(-\frac{1}{2},-\frac{1}{4}\right]$ (4) $\left[-\frac{3}{2},-\frac{5}{4}\right]$

Ans. (2)

Sol. $-\lambda = \sin^4\theta + \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)2 - 2\sin^2\theta \cos^2\theta$

$=1-\frac{4\sin^2\theta\cos^2\theta}{2}$

$=1-\frac{\sin^2 2\theta}{2}$

$\lambda=\frac{\sin^2 2\theta}{2}-1$

$\lambda \in\left[-1,-\frac{1}{2}\right]$

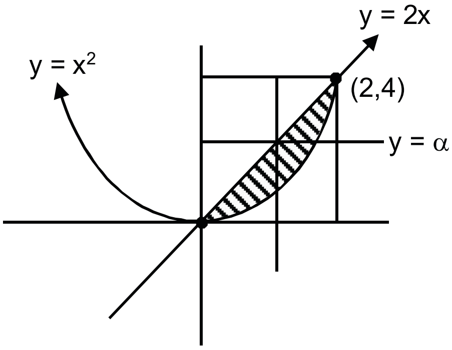
63. Consider a region R = {(x, y) $\in R^2 : x^2 \le y \le 2x$}. If a line y = α divides the area of region R into two equal parts, then which of the following is true ?

(1) $3\alpha^2 - 8\alpha + 8 = 0$ (2) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

(3) $\alpha^3 - 6\alpha^2 + 16 = 0$ (4) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

Ans. (4)

Sol.



$$\int_0^4\left(\sqrt{y}-\frac{y}{2}\right)dy = 2\int_0^\alpha\left(\sqrt{y}-\frac{y}{2}\right)dy$$

$$\Rightarrow \left(\frac{2y^{3/2}}{3}-\frac{y^2}{2}\right)\Bigg|_0^4 = 2\left(\frac{2y^{3/2}}{3}-\frac{y^2}{4}\right)\Bigg|_0^\alpha \Rightarrow \frac{16}{3}-4 = 2\left(\frac{2}{3}\alpha^{3/2}-\frac{\alpha^2}{4}\right)$$

$$\Rightarrow \frac{4}{3} = 2\left(\frac{2}{3}\alpha^{32}-\frac{\alpha^2}{4}\right) \Rightarrow \frac{2}{3} = \frac{8\alpha^{3/2}-3\alpha^2}{12}$$

$$\Rightarrow 8\alpha^{3/2} - 3\alpha^2 = 8$$

64. The set of all possible values of θ in the interval $(0,\pi)$ for which the points (1,2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line x + y = 1 is

(1) $\left(0,\frac{\pi}{2}\right)$ (2) $\left(0,\frac{\pi}{4}\right)$ (3) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$ (4) $\left(0,\frac{3\pi}{4}\right)$

Ans. (1)

Sol. L : x + y – 1 = 0 , A (1,2), B $(\sin\theta, \cos\theta)$

A & B are on same side

$\therefore$ (1 + 2 – 1) $(\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1$

$$\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right) > \frac{1}{2}$$

$$\Rightarrow \theta \in \left(0,\frac{\pi}{2}\right)$$

65. Let f(x) be a quadratic polynomial such that f(–1) + f(2) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in :

(1) (1,3) (2) (–1,0) (3) (–3,–1) (3) (0,1)

Ans. (2)

Sol. Let $f(x) = ax^2 + bx + c$

$f(2) + f(-1) = 0 \Rightarrow 5a + b + 2c = 0$

and $f(3) = 0 \Rightarrow 9a + 3b + c = 0 \Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$

Product of roots $\alpha\beta = \frac{c}{a} = -\frac{6}{5}$

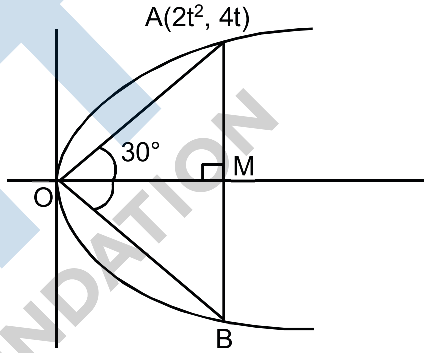
and $\alpha = 3 \Rightarrow \beta = -\frac{2}{5} \in (-1, 0)$

66. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola is :

(1) $64\sqrt{3}$ (2) $192\sqrt{3}$ (3) $128\sqrt{3}$ (4) $256\sqrt{3}$

Ans. (2)

Sol. $y^2 = 8x$, $a = 2$



$A \equiv (2t^2, 2(2)t) \equiv (2t^2, 4t)$

$\tan 30° = \frac{4t}{2t^2} = \frac{2}{t} = \frac{1}{\sqrt{3}}$

$t = 2\sqrt{3}$

Area of $\Delta OAB = 2.\Delta OMA = 2.\frac{1}{2}.(2t^2)(4t) = 8t^3$

$= -8(2\sqrt{3})^3 = 192\sqrt{3}$

67. Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y) \ \forall \ x, y \in R$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k)$, $n \in N$ then the value of n, for which $g(n) = 20$, is

(1) 9 (2) 20 (3) 5 (4) 4

Ans. (3)

Sol. $\because f(x + y) = f(x) + f(y)$

$\therefore f(x) = kx \ \forall \ x \in R$

Now $f(1) = 2 \therefore k = 2$

$\therefore f(x) = 2x \ \forall \ x \in R$

$g(n) = f(1) + f(2) + \ldots\ldots + f(n - 1)$

$= 2[1 + 2 + 3 + \ldots\ldots + (n - 1))]$

$= 2.\frac{(n-1)}{2}.n = 20$

$\Rightarrow n^2 - n - 20 = 0$

$\Rightarrow n = 5$

68. If the sum of first 11 terms of an A.P., $a_1, a_2, a_3 \ldots\ldots$ is 0 ($a_1 \neq 0$), then the sum of the A.P., $a_1, a_3, a_5, \ldots\ldots a_{23}$ is ka_1, where k is equal to :

(1) $\frac{121}{10}$ (2) $-\frac{121}{10}$ (3) $-\frac{72}{5}$ (4) $\frac{72}{5}$

Sol. $\therefore S_{11} = 0 \therefore a + 5d = 0$

Now

$$a_1 + a_3 + a_5 + ... + a_{23} = \frac{12}{2}(2a + (12-1)2d)$$

= 12 (a + 11d)

= 12 (–5d + 11d)

= 72 d

$$= -\frac{75}{5}$$

69. Let n > 2 be an integer. Suppose that there are n Metro stations in a city located around a circular path. Each pair of nearest stations is connected by a straight track only. Further, each pair of nearest station is connected by blue line, whereas all remaining pairs of stations are connected by red line. If number of red lines is 99 times the number of blue lines, then the value of n is

(1) 199 (2) 101 (3) 201 (4) 200

Ans. (3)

Sol. Two consecutive stations = n

Two non consecutive stations = ${}^nC_2 - n$

$${}^nC_2 - n = 99n \quad \Rightarrow \quad \frac{n(n-1)}{5} - n = 99\ n$$

$$\Rightarrow \frac{n^2 - n}{2} = 100\ n \Rightarrow n^2 = 201\ n \Rightarrow n = 201$$

70. Let f : (–1, ∞) → R be defined by f(0) = 1

and $f(x) = \frac{1}{x}\log_e(1+x), x \neq 0$. Then the function f :

(1) increases in (–1,0) and decreases in (0,∞).

(2) decreases in (–1,∞)

(3) decreases in (–1,0) and increases in (0,∞).

(4) increases in (–1,∞)

Ans. (2)

Sol. $$F'(x) = \frac{\frac{x}{1+x} - \ell n(1+x)}{x^2} = \frac{x - (1+x)\ell n(1+x)}{x^2(1+x)}$$

Let g(x) = x – (1 + x) ℓn(1 + x)

⇒ g'(x) = 1 – 1 – ℓn (1 + x)

$$= -\ell n(1+x) \Rightarrow g'(x) = \begin{cases} > 0 & \forall x \in (-1, 0) \\ < 0 & \forall x \in (0, \infty) \end{cases}$$

g_{max} at x = 0 ⇒ g(0) = 0

g(x) < 0 ∀ x ∈ (–1, ∞) ⇒ f'(x) < 0 ∀ x ∈ (–1,∞)

f(x) decreasing ∀ x ∈ (–1,∞)

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

71. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1$ ($\lambda > 0$). If O is the origin and $\overrightarrow{OB}.\overrightarrow{OP}-3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2=6$, then λ is equal to

Ans. (0.8)

Sol. P.V. o pis $\overrightarrow{OP}=\dfrac{\vec{a}+\lambda\vec{b}}{\lambda+1}$

$\because \quad \overrightarrow{OB}.\overrightarrow{OP}-3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2=6$

$\Rightarrow \quad \vec{b}\left(\dfrac{\vec{a}+\lambda\vec{b}}{\lambda+1}\right)-3\left|\vec{a}\times\left(\dfrac{\vec{a}+\lambda\vec{b}}{\lambda+1}\right)\right|^2=6 \Rightarrow \dfrac{\vec{a}.\vec{b}+\lambda\,|\vec{b}|^2}{\lambda+1}-\dfrac{3\lambda^2}{(\lambda+1)^2}\left|\vec{a}\times\vec{b}\right|^2=6$

$\Rightarrow \quad \dfrac{6+\lambda.14}{\lambda+1}-\dfrac{3\lambda^2}{(\lambda+1)^2}.6=6 \qquad \Rightarrow \dfrac{18\lambda^2}{(\lambda+1)^2}+6=6+\dfrac{8\lambda}{\lambda+1}$

$\Rightarrow \quad 18\left(\dfrac{\lambda}{\lambda+1}\right)^2-\dfrac{8\lambda}{\lambda+1}=0 \qquad \left(\dfrac{\lambda}{\lambda+1}\neq 0\right)$

$\Rightarrow \quad 10\lambda = 8 \qquad \Rightarrow \qquad \lambda = 0.8$

72. Let [t] denote the greatest integer less than or equal to t. Then the value of

$\int_1^2 |2x-[3x]|dx$ is

Ans. (1)

Sol. $\int_1^2 |3x-[3x]-x|dx \qquad \Rightarrow \int_1^2 |\{3x\}-x|dx=\int_1^2 (x-\{3x\})dx \Rightarrow \int_1^2 xdx-\int_1^2 \{3x\}dx$

$\Rightarrow \quad \left(\dfrac{x^2}{2}\right)_1^2-3.\int_0^{1/3} 3xdx=\left(\dfrac{4}{2}-\dfrac{1}{2}\right)-9\left(\dfrac{x^2}{2}\right)_0^{1/3} \qquad \Rightarrow \dfrac{3}{2}-\dfrac{9}{2}((1/3)^2-0^2)=1$

73. For a positive integer n, $\left(1+\dfrac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12 , then n is equal to _______

Ans. 118

Sol. $^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2 : 5 : 12$

$\Rightarrow \frac{^nC_r}{^nC_{r-1}} = \frac{5}{2}$ and $\frac{^nC_{r+1}}{^nC_r} = \frac{12}{5}$

$\Rightarrow \frac{n-r+1}{r} = \frac{5}{2}$ and $\frac{n-r}{r+1} = \frac{12}{5}$

$\Rightarrow 2n - 7r + 2 = 0$ and $5n - 17r - 12 = 0$

on solving n = 118 and r = 34

74. If $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$ then $\frac{dy}{dx}$ at $x = 0$ is

Ans. (91)

Sol. $y = \sum_{k=1}^{6} k\cos^{-1}(\cos kx.\cos\alpha - \sin kx.\sin\alpha)$

$= \Sigma k.\cos^{-1}\cos(kx + \alpha)$

$= \Sigma K(kx + \alpha) = \Sigma(k^2x + k\alpha)$

$\frac{dy}{dx} = \sum_{k=1}^{6} k^2 = \frac{6(7)(13)}{6} = 91$

75. If the variance of the terms in an increasing A.P. $b_1, b_2, b_3, \ldots\ldots, b_{11}$ is 90, then the common difference of this A.P. is

Ans. (3)

Sol. Given $b_1, b_2, b_3, \ldots b_{11}$, are in A.P.

$\therefore$ variance of $(b_1, b_2, \ldots b_{11})$ = variance of $(0, d, 2d, \ldots 10d) = 90$

$\Rightarrow \frac{\sum_{i=1}^{11} b_i^2}{11} - \left(\frac{\sum_{i=1}^{11} b_i}{11}\right)^2 = 90$

$\Rightarrow \frac{d^2}{10}.\frac{10.11.21}{6} - d^2.\frac{55}{11}.\frac{55}{11} = 90$

$\Rightarrow 35d^2 - 25d^2 = 90$

$\Rightarrow d^2 = 9$

$\Rightarrow d = 3$